



B.Tech II Year I Semester Supplementary Examinations, July 2022
MATHEMATICS IV

(Common to CE,EEE,ME,ECE,CSE & IT)

Maximum Marks: 70

Date: 19.07.2022 Duration: 3 hours

- Note:**
1. This question paper contains two parts A and B.
 2. Part A is compulsory which carries 20 marks. Answer all questions in Part A.
 3. Part B consists of 5 Units. Answer any one full question from each unit.
 4. Each question carries 10 marks and may have a, b, c, d as sub questions.

Part-A

All the following questions carry equal marks (10x2M=20 Marks)

- 1 Define conjugate Harmonic function
- 2 Find all values of K, such that $f(z) = e^x(\cos ky + i \sin ky)$ is analytic.
- 3 State Cauchy-Integral theorem.
- 4 Write Cross ratio property.
- 5 Define essential singularity, removable singularity.
- 6 Write working rule to evaluate the integrals of the type $\int_0^{2\pi} F(\cos\theta, \sin\theta)d\theta$
- 7 Define periodic function.
- 8 Write Dirichlet conditions for Fourier series expansion.
- 9 State and prove change of scale property.
- 10 Find the Fourier transform of $e^{-ax} \cos ax$

Part-B

Answer All the following questions. (10M X 5=50Marks)

- 11 Find the Bilinear transformation that maps the points -1, i, 1 onto the points 0, i, ∞ . [10]
OR
- 12 Show that the function $u = e^{-2xy} \sin(x^2 - y^2)$ is harmonic, find the conjugate function 'v' and express $u + iv$ as an analytic function of Z. [10]
- 13 Evaluate $\int_C \frac{e^z}{z(1-z)^3}$ where C is (i) $|z| = \frac{1}{2}$ (ii) $|z-1| = \frac{1}{2}$ (iii) $|z| = 2$ [10]
OR
- 14 Expand $f(z) = \frac{1}{z^2 - 3z + 2}$ in the region (i) $0 < |z-1| < 1$ (ii) $1 < |z| < 2$ [10]
- 15 (a) Evaluate $\int_C \frac{\cot hz}{z-i} dz$ where c is $|z| = 2$ [5]
(b) Evaluate $\int_C \frac{\sin \pi z^2 + \cos \pi z^2}{(z-2)(z-1)^2} dz$ where c is $|z| = 2$ [5]
OR
- 16 Evaluate $\int_0^\pi \frac{\cos 2\theta}{1 - 2a \cos \theta + a^2} d\theta$ ($a^2 < 1$) using residue theorem. [10]

- 17 Find the Fourier series for $\begin{cases} x & -\frac{\pi}{2} < x < \frac{\pi}{2} \\ 0 & \frac{\pi}{2} < x < \frac{3\pi}{2} \end{cases}$

Hence deduce that $\frac{1}{1^2} + \frac{1}{3^2} + \frac{1}{5^2} + \dots = \frac{\pi^2}{8}$ [10]

OR

- 18 Find the half-range Cosine series expansion of $f(x) = \sin\left(\frac{\pi x}{l}\right)$ in the range $0 < x < l$. [10]
- 19 Find the finite Fourier sine and Cosine transform, of $f(x) = \sin ax$ in $(0, \pi)$ [10]
- OR
- 20 Find Fourier sine and cosine transform of e^{-ax} and hence deduce the inverse formula. [10]